Dynamic stiffness method and CUF-based component-wise theories applied to free vibration analysis of Composite, Thin-Walled, and Viscoelastic Sandwich Structures

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Abstract

This paper presents a unified analytical formulation for the free vibration analysis of structures with complex cross-sections, including composite, thin-walled, and viscoelastic sandwich structures. The proposed framework combines the Carrera Unified Formulation with Lagrange expansions (CUF(LE)) and the dynamic stiffness method (DSM), and is further extended to damped systems through a complex dynamic stiffness formulation (DDSM), enabling both undamped and damped vibration analyses within a consistent theoretical framework. Within the CUF framework, higher-order three-dimensional displacement fields are expressed using Lagrange polynomials, allowing arbitrary-order expansion of cross-sectional kinematics. A component-wise modeling strategy is adopted, in which different structural parts, such as solid regions, thin-walled components, composite layers, and viscoelastic layers, are treated as independent components and assembled systematically. This enables accurate and flexible modeling of heterogeneous structures with complex geometries and material distributions. In particular, frequency-dependent viscoelastic behavior is incorporated directly into the constitutive relations through complex moduli, ensuring consistent representation of energy dissipation mechanisms. The governing differential equations and natural

boundary conditions are derived using the principle of virtual displacements. An exact dynamic stiffness matrix is established in the frequency domain by relating the amplitudes of harmonically varying loads to the corresponding structural responses. The resulting transcendental eigenvalue problem is solved using the Wittrick–Williams algorithm and its extensions, enabling efficient extraction of both real and complex eigenvalues over a wide frequency range without the need for element refinement. Extensive numerical studies demonstrate that the proposed CUF(LE)–DSM/DDSM framework achieves high accuracy in comparison with three-dimensional finite element solutions while requiring significantly fewer degrees of freedom. The method provides an efficient and versatile tool for broadband vibration analysis and optimization design of advanced structural systems.

Reference：

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